SB-3591/200

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M. Sc. (Part - II) (Applied Mathematics) Examination

March / April – 2011

Mathematical Modeling of Dynamical Systems

Time : 3 Hours] [Total Marks : **70**

Instruction :

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. Sc. Part-2** (APPLIED MATHEMATICS)

Name of the Subject :
☛ **Mathematical Modeling of Dynamical Systems**

☛ Subject Code No. : **3 5 9 1** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

1 (a) Explain the term' Mathematical Modeling',and state its limitations and necessities. **5**

(b) Develop a mathematical model for spring mass system for free undamped motion. Further, solve it analytically. **5**

(c) Describe the technique for formulation of mathemitical modeling. **4**

OR

1 (a) Develop a mathematical model for spring mass system for free damped motion. Further, solve it analytically. **5**

(b) The rate of change of atmospheric pressure p with respect to height h is assumed to be proportional to pressur p. **5**
If pressure values at h=0ft, and h=17,500ft. are respectively p=14.7psi and p=7.35psi, find the pressure at h=10000ft.

(c) Develop a mathematical model for radioactive decay. **4**

2 (a) The concentration of potassium in kidney is found to be 0.0025 mg/cm^3. The kidney is placed in a large vessel in which the potassium concentration is 0.0040mg/cm^3, If in one hour, the concentration in the kidney increases to 0.0027 mg/cm^3, find after how much time will the concentration be 0.0035mg/cm^3. **5**

(b) Given that, the rate of movement of a solute, with respect to time, across a thin membrane is proportional to the area of membrane and the **5**

difference in concentration of the solute on the two sides of the membrane.
Formulate a mathematical model for the given law, and solve it.

(c) A logistic model is defined mathematically by the first order ordinary differential equation. **4**

$$\frac{dN}{dt} = KN(R-N) \qquad (0.1)$$

Show that the solution to (0.1) with N_0 and R,K constaints, is $\frac{N}{R-N} = \frac{N_0}{R-N_0} e^{RKt}$.

OR

2 (a) State the Fick's law of diffusion. Formulate a mathematical model for diffusion defined by Fick's law, and solve it. **5**

(b) Two chemical susbstances combine in the ratio a:b to from a third substance Z. It is known that the amount Z(t) of the third substance at time t consists of the first and second substances in the ratio $\frac{aZ(t)}{a+b} : \frac{bZ(t)}{a+b}$. **5**

Further, the rate of formation of the third substance is proportional to the product of the amount of the two substances, which have not yet combined together.

If the initial amounts of the two substances are A and B, formulate a mathematical model for the given information; and obtain its solution.

(c) Given that the rate of interest determined by a bank is 10% p.a. Find the rate at which the interest is compounded continuously. **4**

3 (a) Show that the motion of a rocket, neglecting gravity and air resistance, is given by $v(t) = u\, ln\left(\frac{m(0)}{m(t)}\right)$, where $m(t)$ = mass of the rocket at time t,and u= velocity of gas relative to the rocket. **5**

(b) Given that the volume of blood in the human body is V and the initial concentration of glucose in the blood stream is c(0). Further, assume that glucose is introduced in the blood stream at a constant rate I. Also, due to physiological needs of the human body, glucose is removed from the blood stream. **5**
Based on the information given above, formulate a mathematical model for diffusion of glucose in the blood. Also, obtain its solution.

(c) Discuss the nature of critical points in the system **4**

defined by $\left.\begin{aligned} x_1' &= 6x_1 - 15x_2 \\ x_2' &= 3x_1 - 6x_2 \end{aligned}\right\}$

OR

3 (a) State the principle of continuity. **5**
Use the principle of continuity to derive a simple compartment model, and solve it.

(b) A patient was given 0.5 micro curies of a type of iodine. Two hours later, it was found that 0.5 micro curies had been taken up by the patients' thyroid. Estimate the amount of iodine taken by the patients' thyroid in two hours; if the patient was given 15 micro curies, initially. **5**

(c) Establish a relation to estimate the doubling, tripling, and quadrupling times for population. **4**

4 (a) A model for single species growth; with time delay is given by the difference equation **5**

$$y_{m+1} - y_m + \alpha y_{m-1} = 0 \qquad (0.2)$$

Solve (0.2) and find the equilibrium points. Also discuss the behaviour of the solution near the equilibrium points.

(b) A mathematical model for the first two species in the ecological system is described by a set of first order ordinary differential equations : **5**

$$\left.\begin{aligned} \frac{dx}{dt} &= xa - bxy \\ \frac{dy}{dt} &= yp - qxy \end{aligned}\right\} (a,b,p,q>0), \qquad (0.3)$$

Where x and y denote the population of species.
Show that solution to (0.3) is $a\ln(y) - by = p\ln(x) - qx + c$
Further, describe the role that each species play in the ecologicl system.

(c) Define the following terms for phase space analysis of a system: **4**
(i) Center
(ii) Attractor
(iii) Focus
(iv) Proper nodel.

OR

4 (a) Derive the two species competition model for the same resources; and solve it. **5**

(b) Given that the price P(t) of commodity at time t changes at rate proportional the differences between the demand D(t) and supply S(t). Formulate a mathematical model for the given details, and solve it. Also, explain the case when $t \to \infty$. **5**

(c) Apply picard's method to solve: **4**

$$x^{\cdot} = -x \;;\; x(0) = 1 \cdot$$

5 (a) A mathematical model is represented by the system of first order ordinary differential equations: **5**

$$\left.\begin{aligned} \frac{dx}{dt} &= x(4-x-y) \\ \frac{dy}{dt} &= y(15-5x-3y) \end{aligned}\right\} (x, y \geq 0). \qquad (0.4)$$

Show that model(0.4) has position of equilibrium; this position is stable, and two species can coexist.

(b) Discuss the stability of the equlibrium positions (0,0) and (p/q , a/b) for the preypredator model represented by the system of first order ordinary differential equations: **5**

$$\left.\begin{aligned} \frac{dx}{dt} &= x(a-by) \\ \frac{dy}{dt} &= y(p-qx) \end{aligned}\right\} (a, b, p, q \geq 0) .$$

(c) Linearize the system (0.5) around its critical points. **4**

$$\left.\begin{aligned} x_1^{\cdot} &= x_2 \\ x_2^{\cdot} &= -x_1^2 + 1 \end{aligned}\right\} \qquad (0.5)$$

OR

5 (a) The velocity field for the motion of a car is given by **5**

$$u(x,t) = \frac{30(x+L)}{15t+L}; \text{ L constant.}$$

Determine the motion X(t) of car which starts at x=L/2 at time t=0.

(b) The traffic is moving at 10 mph, so that cars are at a distance of 'one car length'. Describe the traffic flow. **5**

(c) For a given n×n matrix A, show that **4**

$$e^{P-1AP} = P^{-1}e^{A}P,$$

where P is a matrix whose columns are eigen vectors of matrix A.
